

Deflection

معادلة العزم

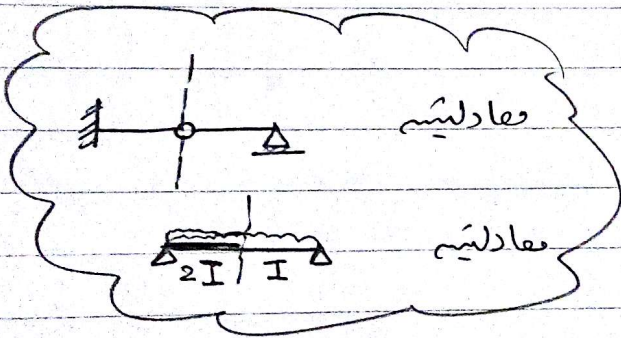
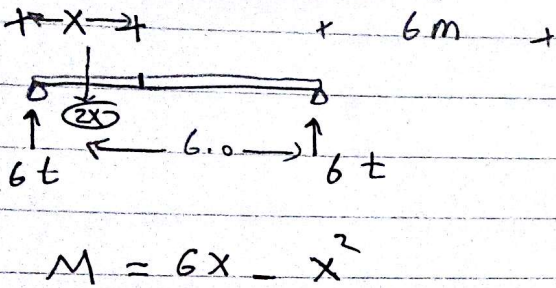
اول طريقة

* Double Integration :-

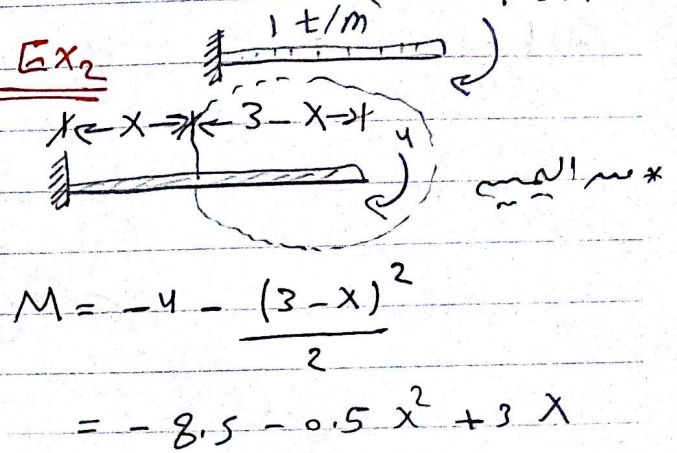
$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

* لقسم W يجب معادلة العزم *

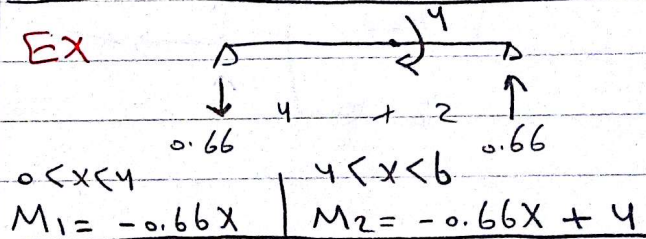
Ex1



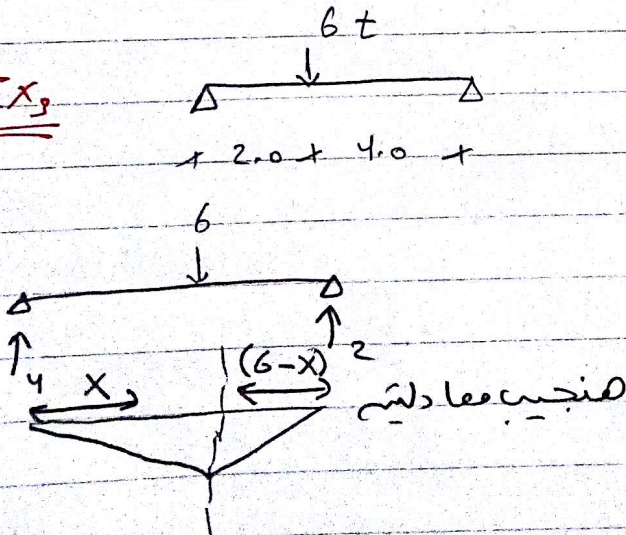
Ex2



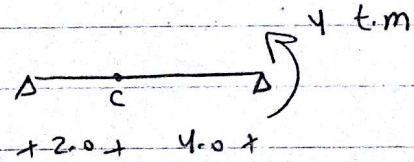
Ex



Ex3



Ex4



① $0 < x < 2 \Rightarrow M = 4x$

② $2 < x < 6 \Rightarrow M = 12 - 2x$

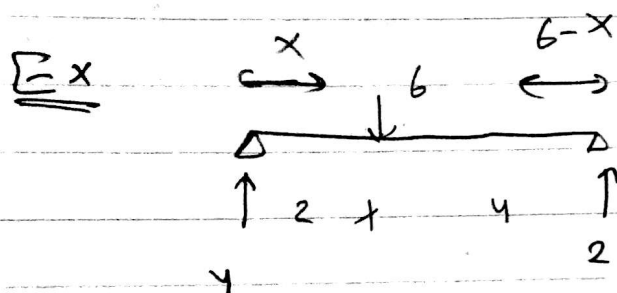
* boundary condition نستخرج من لوابت



$$x=0 \rightarrow y=0$$



$$x=0 \quad \bar{y}=0$$



$$0 < x < 2 \quad M_1 = 4x$$

$$2 < x < 4 \quad M_2 = 12 - 2x$$

$$\bar{y}_1 = 2x^2 + C_1$$

$$y_1 = \frac{2}{3}x^3 + C_1x + C_2 \rightarrow C_2 = 0.0$$

$$\bar{y}_2 = 12x - x^2 + C_3$$

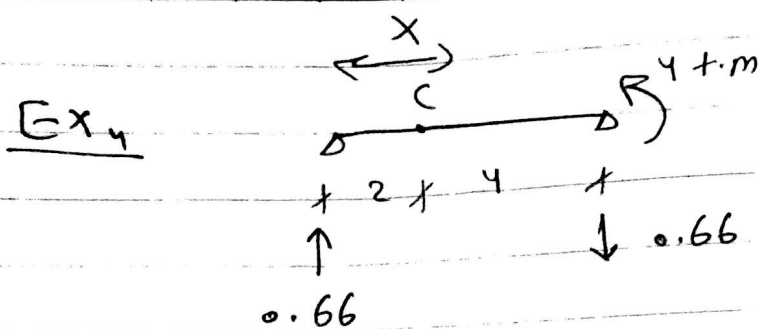
$$y_2 = 6x^2 - \frac{x^3}{3} + C_3x + C_4 \rightarrow (C_3, C_4)$$

عنای نقطه ل Deformation, Δ

$x=2 \quad \bar{y}_1 = \bar{y}_2 \quad \ominus \quad \Delta \quad \Delta \quad \Delta$

$\rightarrow C_3$

$$x=2 \quad y_1 = y_2$$



$$EI = 4000 \text{ t.m}^2$$

find $y(x)$

$\times y_{max}$

$$M = 0.66x$$

$$\bar{y} = - \frac{0.66x}{EI}$$

$$\bar{y} = \frac{1}{EI} \left[-0.66 \frac{x^2}{2} + C_1 \right]$$

$$y = \frac{1}{EI} \left[-0.66 \frac{x^3}{6} + C_1 x + C_2 \right]$$

$$\text{at } x = 0.0 \quad y = 0.0 \rightarrow C_2 = 0$$

$$\text{at } x = 6.0 \quad y = 0.0 \rightarrow C_1 = 4.0$$

$$\times y_c = \frac{1}{EI} \left[-0.66 \times \frac{2^3}{6} + 4 \times 2 \right] = 1.77 \times 10^{-3} \text{ m}$$

$$\bar{y}_c = \frac{1}{EI} \left[-0.66 \frac{2^2}{2} + 4 \right] = 6.6 \times 10^{-4} \text{ rad}$$

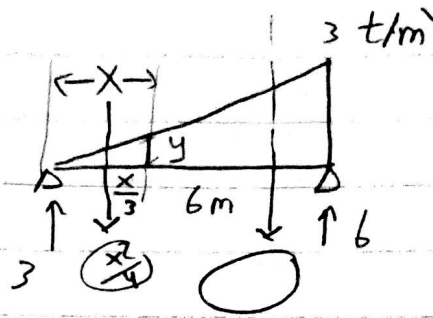
$$\times y_{max} \rightarrow \text{(rotation) slope is } \bar{y} = 0$$



$$0.0 = \frac{1}{4000} \left[-0.66 \frac{x^2}{2} + 4 \right] \rightarrow x = 3.4 \text{ m}$$

$$y_{max} = \frac{9.28}{EI} \text{ m}$$

Ex

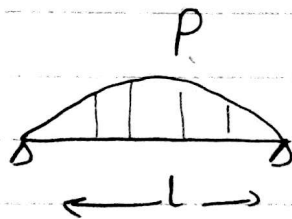


$$\frac{y}{x} = \frac{3}{6}$$

$$y = \frac{3x}{6} = \frac{1}{2}x$$

$$M = 3x - \left(\frac{x^2}{4} \cdot \frac{x}{3} \right)$$

Ex



$$P = P_0 \sin\left(\frac{\pi x}{L}\right)$$

$$Q = \int -P dx = P \left(\frac{L}{\pi} \right) \cos\left(\frac{\pi x}{L}\right) + C_1$$

$$M = \int Q dx = P \frac{L^2}{\pi^2} \sin\left(\frac{\pi x}{L}\right) + C_1 x + C_2$$

at $x=0$ $M=0 \rightarrow C_2=0$

at $x=L$ $M=0 \rightarrow C_1=0$

$$\bar{y} = \frac{1}{EI} \left[P_0 \left(\frac{L}{\pi} \right)^3 \cos\left(\frac{\pi x}{L}\right) \right]$$

$y \rightarrow \bar{y}=0$

$$\cos \frac{\pi x}{L}$$

$\cos x = 0$ (مساوي)

① $x = \pi/2$

② $x = \frac{3\pi}{2}$

① - $\frac{\pi x}{L} = \pi/2$

$x = \frac{L}{2}$

② - $\frac{\pi x}{L} = \frac{3\pi}{2}$ $x = \frac{3L}{2}$ مرفوض