**VALLIAMMAI ENGINEERING COLLEGE**
**SRM Nagar, Kattankulathur**

## Department of Information Technology

## Question Bank- Even Semester 2014-2015

**IV Semester**

# MA6453 – Probability and Queueing Theory

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### UNIT - I - RANDOM VARIABLES

### PART-A

1. A continuous RV X has PDF f(x) = 3 $x^2$ , 0 <x<1 , 0 otherwise. Find k such that P ( X >k) = 0.5
2. If X and Y are independent random variable with variance 2 and 3. Find the variance of 3X+4Y.
3. Define random variable
4. Define Geometric distribution
5. Find the moment generating function of binomial distribution
6. The probability that a man shooting a target is ¼. How many times must he fire so that the probability of his hitting the target at least once is more than 2/3.
7. Find C, if P[X=n]=C$\left(\frac{2}{3}\right)^n$; n=1,2,3…….
8. The CDF of a continuous RV is given by F ( x) = $\begin{cases} 0 & x < 0 \\ 1 - e^{-\frac{x}{5}} & x > 0 \end{cases}$ Find the PDF and mean of X
9. Establish the memoryless property of the exponential distribution
10. Write two characteristics of the Normal Distribution
11. Find the mean of the Poisson distribution which is approximately equivalent to B(300, 0.2).
12. The mean and variance of the binomial distribution are 4 and 3 respectively. Find P(X=0)
13. Show that the moment generating function of the uniform distribution f(x) =1 / 2a , -- a < x < a, about origin is sinh(at) / at
14. If a random variable X has the MGF $M_X(t)=\frac{2}{2-t}$ . Find the standard deviation of X.
15. If 3% of the electric bulb s manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.
16. If X is aPoisson variate such that P[X=1]=3/10 and P[X=2]=1/5 .Find P[X=0] and P[X=3]
17. Let the random variable X denote the sum obtained in rolling a pair of dice. Determine the probability mass function of X.
18. The number of hardware failures of a computer system in a week of operations has the following pmf:

| No.of failures | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Probability | .18 | .28 | .25 | .18 | .06 | .04 | .01 |

Find the maean of the number of failures in a week.

19. If X is uniformly distributed in$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.Find the pdf of Y = tanx
20. Define Poisson distribution and state any two instances where Poisson distribution may be successfully employed

## PART-B

1. (a)(i)Define the moment generating function (MGF) of a random variable ? Derive the MGF, mean, variance and the first four moments of a Gamma distribution.
(ii)Describe Binomial B (n, p) distribution and obtain the moment generating function. Hence compute (i) the first four moments and (ii)the recursion relation for the central moments.
(b)(i)A random variable X has the following probability distribution.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(x): | 0 | K | 2K | 2K | 3K | $K^2$ | $2 + K^2$ | $7K^2+K$ |

Find A. The value of K
B. P( 1.5<X<4.5/X>2) and
C. The smallest value of n for which P (X≤n)>1/2

(ii)Find the MGF of a random variable X having the pdf $f(x) = \begin{pmatrix} \frac{x}{4e^{\frac{x}{2}}} & x > 0 \\ 0 & otherwise \end{pmatrix}$. Also deduce the first four moments about the origin.

2. (a)(i)Derive the MGF of Poisson distribution and hence or otherwise deduce its mean and variance.
(ii)Describe gamma distribution. Obtain its moment generating function . Hence compute its mean and variance.
(b)(i)Suppose that customers arrive a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min
A. Exactly 4 customers arrive and
B. more than 4 customers arrive
(ii) If X and Y are independent RVs each normally distributed with mean zero and variance $\sigma^2$ , find the pdf of $R = \sqrt{x^2 + y^2}$ and $\emptyset = \tan^{-1}\left(\frac{Y}{X}\right)$

3. (a)(i)A random variable X has pdf $f_X(x) = \begin{Bmatrix} kx^2 e^{-x} & x > 0 \\ 0 & otherwise \end{Bmatrix}$ . Find the r$^{th}$ moment of X about origin. Hence find the mean and variance.
(ii)A random variable X is uniformly distributed over (0,10) . Find (1) P(X<3) , P(X<7) and P(2<X<5) (2) P(X=7).

(b)(i)An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.
(1)What is the probability that all four phones are busy?
(2)What is the probability that at least two of them are busy?
(ii)Given that X is distributed normally, if P[X<45]=0.31 and P[X>64]=0.08 , find the mean and standard deviation of the distribution.

4. (a)(i)The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$
A. What is the probability that the repair time exceeds 2 hours?
B. What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours?
(ii)The probability density function of a random variable X is given by

$$f_X(x) = \begin{Bmatrix} x & 0 < x < 1 \\ k(2-x) & 1 \le x \le 2 \\ 0 & otherwise \end{Bmatrix}$$

(i)Find the value of k
(ii) P(0.2<x<1.2)
(iii) What is P[0.5<x<1.5/x≥1]
(iv) Find the distribution function of f(x).
(b)(i)The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. It 3 students are selected at random from this set, what is the probability that exactly 2 of them will have marks over 70?

(ii)The chances of three candidates A, B and C to become the manager of a company are in the ratio 3:5:4. The probability of introducing a special bonus scheme by them if selected are 0.6,0.4, and 0.5 respectively. If the bonus scheme is introduced, what is the probability that B has become the manager?

5. (a)(i)Find the MGF , mean and variance of the Binomial distribution

(ii)There are three unbiased coins and one biased coin with head on both sides. A coin chosen at random and tossed 4 times., what is the probability that the biased coin has been chosen?

(b)(i)A wireless set is manufactured with 25 soldered joints each. On an average one joint in 500 is defective. How many sets can be expected to be free from defective joints in a consignment of 10,000 sets?

(ii)Find the MGF of the Normal distribution.

6. (a)(i)The daily consumption of bread in a hostel in excess of 2000 loaves is approximately distributed as Gamma variable with parameter k=2 and $\lambda = \frac{1}{1000}$ . The hostel has a daily stock of 3000 loaves. What is the probability that the stock is insufficient on a day?

(b)(i) Let X be a continuous random variable with p.d.f

$$f(x) = \begin{Bmatrix} ax\,,\ 0 \le x \le 1 \\ a\ \ ,\ \ 1 \le x \le 2 \\ -ax + 3a,\ \ 2 \le x < 3 \\ 0,\quad otherwise \end{Bmatrix}$$

(i)Determine the constant a (ii) Compute P(X≤ 1.5) (iii) The c.d.f of X.

(ii)The probability function of a random variable x is given by $P[X = x] = \frac{1}{2^x}$ , x=0,1,2………

Find (i)P[X is even]

(ii) P[X is odd]

(iii)P[X≥ 5]

(iv)P[X is divisible by 3]

7. (a)(i)State and prove memoryless property for exponential distribution

(ii)State and Prove memoryless property for Geometric distribution.

(b)(i)Derive the poisson distribution as limiting form of binomial distribution

(ii)The number of monthly breakdown of a computer is a random variable having a poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month

(i) Without breakdown

(ii) With only one breakdown and

(iii)With atleast one breakdown

8.(a)(i) A die is tossed until 6 appears. What is the probability that it must be tossed more than 5 times?

(ii)The diameter of an electric cable, say X, is assumed to be a continuous r.v with p.d.f $f(x) = 6x(1-x)\ \ o < x < 1$

(i)Check that the above is a p.d.f.

(ii)Determine a and b such that P[X<b]=P[X>b]

(iii)Find the distribution function of X

(b)(i) The monthly demand for Alwyn watches is known to have the following probability distribution

| Demand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Probability | 0.08 | 0.12 | 0.19 | 0.24 | 0.16 | 0.10 | 0.07 | 0.04 |

Determine the expected demand for watches.Also compute the variance.

(ii) The density function of a random variable X is given by $f(x) = k\,x\,(2 - x)$, $0 \le x \le 2$. Find K, mean, variance and r $^{th}$ moment.

9. (a)(i) State and explain the properties of Normal N ( $\mu,\sigma^2$ ) distribution

(ii) Out of 2000 families with 4 children each , how many would you expect to have i) at least 1 boy ii) 2 boys iii) 1 or 2 girls iv) no girls

(b) (i) Define Gamma distribution .Prove that the sum of independent Gamma variates is a Gamma variable

(ii) Find the first four central moments of normal distribution

10. (a)(i) VLSI chips , essential to the running condition of a computer system, fail in accordance with a Poisson distribution with the rate of one chip in about 5 weeks .if there are two spare chips on hand ,and if a new supply will arrive in 8 weeks . What is the probability that during the next 8 weeks the system will be down for a week or more, owing to a lack of chips?

(ii) The probability mass function of a RV X is defined as $P(X=0) = 3C^2$, $P(X=1) = 4C-10C^2$, $P(X=2) = 5C-1$ where C >0,and $P(X=r) = 0$ if $r \ne 0,1,2$.Find 1)The value of C 2) P ( 0<X <2 / X >0 ) 3) The distribution function of X 4) The largest value of X for which F(x) < ½

(b)(i) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8.What is the probability that he will finally pass the test i) on the fourth trail and ii) in less than 4 trials.

(ii) Find the moment generating function of the random variable X having the pdf

$$f(x) = \begin{cases} \frac{1}{2}\, x^2\, e^{-x}, & x > 0 \\ 0,, & otherwise \end{cases}$$ Find E(X) .

**UNIT-2 TWO DIMENSIONAL RANDOM VARIABLES**

**PART-A**

1. State the basic properties od joint distribution of (X,Y) when X and Y are random variables.
2. The joint probability density function of the random variable (X,Y) is given by
$f(x,y) = Kxye^{-(x^2 + y^2)}$, $x > 0$, $y > 0$ Find the value of K
3. Given the RV X with density function
$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$ Find the pdf of $y=8x^3$
4. The joint pdf of a random variable (X,Y) is $f_{xy}(x,y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2$, $0 \le y \le 1$. Find P(X<Y)
5. Let X and Y be two independent RVs with joint pmf $P(X=x, Y=y) = \begin{cases} \frac{x+2y}{18}, & x = 1,2,\ y = 1,2 \\ 0, & otherwise \end{cases}$

Find the marginal probability mass function of X and E (X)

6. Find the acute angle between the two lines of regression, assuming the two lines of regression
7. If X and Y are random variables having the joint density function
$f(x,y) = 1/8\,(6 - x - y)$, $0 < x < 2$, $2 < y < 4$, find P(X+Y < 3 )
8. Distinguish between correlation and regression
9. The equation of two regression lines obtained by in a coreelation analysis is as follows :
$3x + 12y = 19$, $3y + 9x = 46$.Obtain the correlation coefficient 2.Mean value of X and Y
10. Let X and Y be integer valued random variables with $P(X=m, Y=n) = q^2p^{m+n-2}$, n, m = 1,2,… and p+q = 1. Are X and Y independent ?
11. . Let X and Y be random variables with joint density function

$$f_{XY}(x,y) = \begin{cases} 4xy\ ,\ 0 < x < 1\ ,\ 0 < y < 1 \\ 0,\ otherwise \end{cases}$$ find E(XY)

12. Prove that Cov(X,Y)=E(XY)-E(X)E(Y)
13. The correlation coefficient of two dimensional random variable X and Y is -1/4 while this variances are 3 and 5. Find the covariance.
14. Let (X,Y) be a two dimensional random variable . Define Covariance of (X,Y). If X and Y are Independent, what will be the covariance of (X,Y)?
15. The tanget of the angle between the lines of regression of Y on X and X on Y is 0.6 and $\sigma_X = \frac{1}{2}\sigma_Y$ , find the correlation coefficient between X and Y.
16. If Y=-2X+3, find the COV(X,Y)
17. Define joint pdf of two RVs X and Y and state its properties
18. If X and Y have joint pdf $f(x,y) = \begin{cases} x + y\ ,\ 0 < x < 1,\ 0 < y < 1 \\ 0,\ otherwise \end{cases}$ check whether X and Y are independent.
19. Can the joint distributions of two random variables X and Y be got if their marginal distributions are known?
20. If X has mean 4 and variance 9 while Y has mean -2 and variance 5 and the two are independent Var ( 2X + Y -5 )

**PART-B**

1.(a)(i) The If the joint pdf of a two dimensional random variable (X,Y) is given by

$f(x,y) = \begin{cases} x^2 + \frac{xy}{3} & 0 < x < 1,\ 0 < y < 2 \\ 0, elsewhere \end{cases}$ Find (i) P(X > ½ ) (ii) P(Y<X) and (iii) P(X+Y≥1) and (iv)Find the conditional density function

(ii)The joint p.d.f of the random variable (X,Y) is f(x,y)=3(x+y) 0≤x≤1, 0≤y≤1, x+y≤1, find Cov(X,Y).

(b)(i)Marks obtained by 10 students in Mathematics (x) and statistics (y) are given below

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 60 | 34 | 40 | 50 | 45 | 40 | 22 | 43 | 42 | 64 |
| y | 75 | 32 | 33 | 40 | 45 | 33 | 12 | 30 | 35 | 51 |

Find the two regression lines. Also find y when x=55.

(ii)If X and Y are independent RVs with pdf's $e^{-x}; x \geq 0$ and $e^{-y}; y \geq 0$, respectively, find the pdf of $U = \frac{X}{X+Y}$ and V=X+Y . Are U and V independent?

2. (a)(i) The joint probability mass function of (X, Y) is given by p(x,y) = K (2x+3y) , x = 0,1,2; y = 1,2,3. Find all the marginal and conditional probability distributions.Also find the Probability distribution of (X+Y).

(ii)Two independent random variables X and Y are defined by $f_X(x) = \begin{Bmatrix} 4ax: & 0 < x < 1 \\ 0: otherwise \end{Bmatrix}$

And $f_Y(y) = \begin{Bmatrix} 4ay: & 0 < x < 1 \\ 0: otherwise \end{Bmatrix}$ Show that U=X+Y and V=X-Y are correlated.

(b)(i) The equation of two regression lines obtained by in a coreelation analysis is as follows : 3x + 12 y = 19 , 3y + 9x =46.Obtain the correlation coefficient 2.Mean value of X and Y

(ii) Given f(x,y) = cx(x-y) , 0<x<2 ,-x<y<x 1)Evaluate C 2) Find f(x) 3) F(y/x) 4) f(y).

3. (a)(i)The joint probability density function of the random variable (X,Y) is given by

$f(x,y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$ . Find the value of K and Cov (X,Y). Are X and Y independent?

(ii)If X and Y are uncorrelated random variable with variances 16 and 9. Find the correlation co-efficient between X+Y and X-Y.

(b)(i)Let (X,Y) be a two dimensional random variable and the probability density function be given by $f(x,y) = x + y,\ 0 \le x, y \le 1$. Find the P.d.f of U=XY.

(ii) The regression equation of X on Y is 3Y-5X+108=0. If the mean value of Y is 44 and the variance of X is 9/16th of the variance of Y. Find the mean value of X and the correlation coefficient.

4. (a)(i) If X and Y are independent RVs with density function f(x) = 1 ,1 <x<2 , 0 otherwise and f (y) =y / 6 , 2 < y<4 , 0 otherwise .Find the density function of Z =XY.

(ii)Two dimensional random variable X and Y the joint pdf $f(x,y) = \begin{cases} K(4-x-y), 0 \le x \le 2, 0 \le y \le 2 \\ 0, otherwise \end{cases}$ Find the correlation coefficient between X and Y.

(b)(i)Let X and Y be jointly distributed with p.d.f $f(x,y) = \begin{cases} \frac{1}{4}(1+xy), & |x| < 1,\ |y| < 1 \\ 0, otherwise \end{cases}$

Show that X and Y are not independent.

(ii) If the joint pdf of X and Y is given by f (x,y) = $e^{-(x+y)}$ x>0 ,y>0 0 , elsewhere.

Find 1.The marginal pdf of X and Y 2.Are they independent . 3. P ( X>2, Y <4). 4. P (X >Y)

5.(a)(i) Given the joint probability density f(x,y) = $\begin{cases} \frac{2}{3}(x+2y) for \ 0 < x < 1, 0 < y < 1 \\ 0, \qquad otherwise \end{cases}$

Find the marginal densities, conditional density of X given Y=y and P(X<1/2 /Y= ½)

(ii)The two dimensional random variable (X,Y) has the joint probability mass function $f(x,y) = \frac{x+2y}{27}$, x=0,1,2; y=0,1,2

(i)Find the conditional distribution of Y given X=x

(ii)Also find the conditional distribution of Y given X=1.

(b)(i)Three balls are drawn at random without replacement from a box containing 2 white, 3 red, and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn , find the joint probability distribution of (X,Y).

(ii) The joint probability mass function of X and Y is

| P(x,y) | | 0 | 1 | 2 |
|---|---|---|---|---|
| X | 0 | 0 . 1 | 0 . 04 | 0 . 02 |
| | 1 | 0 . 08 | 0 . 2 | 0 . 06 |
| | 2 | 0 . 06 | 0 . 14 | 0 . 3 |

Compute the marginal PMF of X and of Y, P [ X ≤ 1, Y ≤ 1] and check if X and Y are independent .

6. (a)(i) Let the jointprobability distribution of X and Y be given by

| y | -1 | 0 | 1 |
|---|---|---|---|
| -1 | 1/6 | 1/3 | 1/6 |
| 0 | 0 | 0 | 0 |
| 1 | 1/6 | 0 | 1/6 |

Show that their covariance is zero even though the two RVs are not independent

(ii) The RVs X and Y are statistically independent having a gamma distribution with parameters ( m ,1/2) and ( n,1/2 ) respectively.Derive the probability density function of a RV U = X / X+Y

(b)(i) Random variables X and Y have the joint distribution ( when p + q = 1 , 0 < p < 1 and λ >0 )

p( x , y) = $\frac{e^{-\lambda}\lambda^{x}}{y!}\,\frac{p^{y}\, q^{x-y}}{(x-y)!}$ y = ,1,2,3,…x; x=1,2,3,…Find marginal and conditional distribution and evaluate P(X=1)

(ii)If X and Y are two random variables having joint density function

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}: & 0 < x < 2, 2 < y < 4 \\ 0: otherwise \end{cases}$$ Find (i) P(X<1∩ $Y < 3$) (ii)P(X<1/Y<3)

(iii)P(X+Y<3)

7. (a)(i) Given the joint p.d.f of X and Y is $f(x,y) = \begin{cases} 8xy; & 0 < x < y < 1 \\ 0; otherwise \end{cases}$

Find the marginal and conditional p.d.f's of X and Y. Are X and Y independent?

(ii)The random variable (X,Y) has the joint p.d.f $(x,y) = \begin{cases} (x+y): & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0: elsewhere \end{cases}$

Compute r(X,Y).

(b)(i) Two random variable X and Y are defined with Y=4X+9. Find the correlation coefficient between X and Y.

(ii) If X and Y are standardized random variables and r(aX+bY, bX+aY)=$\frac{1+2ab}{a^2+b^2}$ find r(X,Y), The coefficient of correlation between X and Y.

8.(a)(i) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible Variance of X=9 Regression equations 8x-10y+66=0, 40x-18y=214 Find (i) The mean values of X and Y.(ii) The standard deviation of Y. (iii)The co-efficient of correlation between X and Y.

(ii)Let X and Y be random variables with joint p.d.f

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), & 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$$

Find 1. The correlation coefficient $r_{xy}$

2. The two regression curves.

(b)(i) The joint pdf of the random variable is given by f(x,y) = $e^{-(x+y)}$, for x ≥ 0, y ≥ 0. Find the pdf of U = $\frac{X+Y}{2}$

(ii) From the following data find

(1)The two regression equations

(2) The coefficient of correlation between the marks in Mathematics and Statistics

(3) The most likely marks in Statistics when marks in Mathematics are 30

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks in Maths : | 25 | 28 | 35 | 32 | 31 | 36 | 29 | 38 | 34 | 32 |
| Marks in Staitistics: | 43 | 46 | 49 | 41 | 36 | 32 | 31 | 30 | 33 | 39 |

9. (a)(i) Calculate the Karl-Pearson's coefficient of correlation from the following data

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| X : | 39 | 65 | 62 | 90 | 82 | 75 | 25 | 98 | 36 | 78 |
| Y : | 47 | 53 | 58 | 86 | 62 | 68 | 60 | 91 | 51 | 84 |

(ii) If two dimensional RV X and Y are uniformly distributed in 0 <x<y<1 find 1)The correlation coefficient $r_{xy}$ 2)Regression equations

(b)(i) If two dimensional RV X and Y are uniformly distributed in 0 <x<y<1 find 1)The correlation coefficient $r_{xy}$ 2)Regression equations

(ii) If f(x,y) $=\dfrac{6-x-y}{8}$, $0 \le x \le 2$, , $2 \le y \le 4$ for a bivariate random variable (X,Y), find the correlation coefficient $\rho_{xy}$

10. (a)(i) Find the correlation between X and Y if the joint probability density of X and Y is f(x,y)= 2 for x>0, y >0,x+y<1

(ii) If X and Y are independent RVs , show that the pdf of U = X + Y is given by h(u) = $\int_{-\infty}^{\infty} f(v)f(u-v)dv$

(b)(i)The joint P.d.f of two random variables X and Y is given by $f(x,y)=\frac{9(1+x+y)}{2(1+x)^4(1+y)^4}$ $0 \le x < \infty,\ 0 \le y \le \infty$ Find the marginal distributions of X and Y and the conditional distributions of y for X=x.

(ii)If the joint p.d.f of (X,Y) is given by f(x,y)=2, 0<x<y<1. Find

(i)Marginal density functions of X and Y.

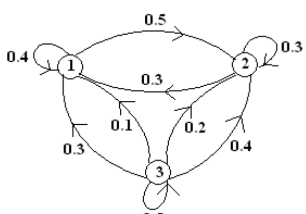
(ii) Conditional densities f(x/y) and f(y/x)

(iii)Conditional variance of X given Y=1/2.

## UNIT - III - MARKOV PROCESSES AND MARKOV CHAINS

## PART – A

1. What is a Markov process?
2. Define (i) a stationary process (ii) wide sense stationary process
3. Define Ergodic process.
4. Define Poisson process.
5. State the properties of Poisson process.
6. Define accessible states, communicate and irreducible Markov chain.
7. Consider the Markov chain with 2 states and transition probability matrix $P=\begin{bmatrix}\frac{3}{4} & \frac{1}{4}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$. Find the stationary probabilities of the chain.
8. The one-step transition probability matrix of a Markov chain with states (0,1) is given by $P=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. Is it irreducible Markov chain?
9. Find the transition matrix of the following transition diagram.




10. Prove that the random process $X(t)=A\cos(\omega_c t+\theta)$ is not stationary if it is assumed that A and $\omega_c$ are constants and θ is a uniformly distributed variable on the interval (0,π).
11. Prove that a first order stationary random process has a constant mean.
12. Find the mean and variance of a stationary random process whose auto correlation function is given by (i) $R_{XX}(\tau)=18+\dfrac{2}{6+\tau^2}$ (ii) $R_{(Z)}=\dfrac{25Z^2+36}{6.25Z^2+4}$

13. Check whether the Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is irreducible or not?
14. Consider the random process $\{X(t), X(t) = \cos(t + \phi)\}$ where $\phi$ is uniform in $\left(-\pi/2, \pi/2\right)$. Check whether the process is stationary.
15. If X(t) and Y(t) are two wide – sense stationary random processes and $E\{|X(0) - Y(0)|^2\} = 0$, prove that $R_{XX}(\tau) = R_{XY}(\tau) = R_{YY}(\tau)$.
16. Define continuous random process and discrete random process. Give an example.
17. A random process X (t) = A sin $t$ + B cos $t$ where A and B are independent random variables with zero means and equal standard deviations. Show that the process is stationary of the second order.
18. When is a Markov chain, called Homogeneous?
19. Define renewal process and give the example.
20. Determine which of the following are stochastic matrix

$$A = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \end{bmatrix} \quad B = \begin{bmatrix} 3/4 & 3/4 \\ 2/3 & 2/3 \end{bmatrix} \quad C = \begin{bmatrix} 3/2 & -1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

**PART – B**

1. a) The process {X(t)} whose probability distribution under certain conditions is given by

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2 \\ \dfrac{at}{(1+at)}, & n = 0 \end{cases}$$

Show that it is not stationary.

b) Two random processes X(t) and Y(t) are defined by $X(t) = A\cos\omega t + B\sin\omega t$ and $Y(t) = B\cos\omega t - A\sin\omega t$. Show that X (t) and Y(t) are jointly wide – sense stationary if A and B are uncorrelated random variables with zero means and the same variances and ω is constant.

2. a)Given that the random process $X(t) = \cos(t + \varphi)$ where $\varphi$ is a random variable with density function $f(x) = \frac{1}{\pi}, \frac{-\pi}{2} < \varphi < \frac{\pi}{2}$ .Check whether the process is stationary or not.

b) The transition probability matrix of a Markov chain $\{X_n\}$, n = 1,2,3, …. having 3 states 1,2 and 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$ and the initial distribution is P(0) = (0.7, 0.2, 0.1)

Find i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

3. a) Show that the random process $X(t) = A\sin(\omega t + \theta)$ is wide-sense stationary process where A and ω are constants and θ is uniformly distributed in (0, 2π).

b) On a given day, a retired English professor, Dr. Charles Fish amuses himself with only one of the following activities reading (i), gardening (ii) or working on his book about a river valley (iii), for $1 \le i \le 3$, let $X_n = i$, if Dr. Fish devotes day $n$ to activity $i$. Suppose that $\{X_n : n=1,2\ldots\}$ is a

Markov chain, and depending on which of these activities on the next day in given by the t. p. m $P = \begin{bmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{bmatrix}$ Find the proportion of days Dr. Fish devotes to each activity.

4. a) The number of demands of a cycle on each day in a cycle hiring shop is Poisson distributed with mean 2. The shop has 3 cycles. Find the proportion of days on which (i) no cycle is used (ii) some demand of cycles is refused.
   b) Three boys A, B and C are throwing a ball to each other. *A* always throws the ball to *B* and *B* always throws the ball to *C* but *C* is just as likely to throw the ball to *B* as to *A*. Show that the process is Markovian. Find the transition matrix and classify the states.

5. a) Consider a random process $X(t) = B\cos(50t + \Phi)$ where B and Φ are independent random variables. B is a random variable with mean 0 and variance 1. Φ is uniformly distributed in the interval [-π,π]. Find the mean and auto correlation of the process.
   b) Let $\{X_n : n = 1,2,3 \ldots\ldots\}$ be a Markov chain on the space S = {1,2,3 } with one step t.p.m

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$$

   i) Sketch the transition diagram
   ii) Is the chain irreducible? Explain.
   iii) Is the chain ergodic? Explain.

6. a) Show that the random process $X(t) = A\cos(\omega + \theta)$ is wide sense stationary if A and ω are constant and is a uniformly distributed random variable in (0, 2π).
   b) (i) Prove that a Poisson Process is a Markov chain.
   (ii) Prove that the difference of two independent Poisson process is not a Poisson process.
   (iii) Prove that the sum of two independent Poisson process is a Poisson process.
   (iv) Find the mean and autocorrelation of the Poisson processes.

7. a) Given a random variable Y with characteristic function $\Phi(\omega) = E\left[e^{j\omega Y}\right]$ and a random process defined by $X(t) = \cos(\lambda t + y)$, show that $\{x(t)\}$ is stationary in the wide sense if $\Phi(1) = \Phi(2) = 0$
   b) If the customers arrive in accordance with the Poisson process, with rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 minute, (ii) between 1 and 2 minutes, (iii) less than 4 minutes.

8. a) Derive the balance equation of the birth and death process.
   b) A man either drives a car pr catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.
9. a) A fair dice is tossed repeatedly. If $X_n$ denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Find also $P\{X_2=6\}$ and $P^2$.
   b) An engineer analyzing a series of digital signals generated by attesting system observes that oly 1 out of 15 highly distorted signal with no recognizable signal whereas 20 out of 23 recognized signals follow recognizable signals with no highly distorted signals between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted.

10. a) The t.p.m of a Markov chain $\{X_n\}$, n= 1, 2,3….. having 3 states 1,2, and 3 is. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

Find the nature of states of t.p.m.

b) A salesman's territory consists of three regions A, B, C. He never sells in the same region on successive days. If he sells in region A, then the next day he sells in B. However, if he sells either B or C, then the next day he is twice as likely to sell in A as in the other region. How often does he sell in each of the regions in the steady state?

## UNIT - IV - QUEUEING THEORY

### PART - A

1. State the characteristics of a queueing model.
2. What are the service disciplines available in the queueing model?
3. Define Little's formula.
4. For (M/M/1) : ( $\infty$/FIFO) model, write down the Little's formula.
5. Consider an M/M/1 queueing system. Find the probability of finding at least *n* customers in the system.
6. What do you mean by transient and steady state queueing systems?
7. Write down the formula for average waiting time of a customer in the queue for (M/M/F) : (K/FIFO).
8. What is the probability that a customer has to wait more than 15 *min* to get his service completed in a (M/M/1) : ( $\infty$/FIFO) queue system, if $\lambda$=6 *per hour* and $\mu$= 10 *per hour?*
9. If $\lambda$= 3 per hour, $\mu$= 4 per hour and maximum capacity K = 7 in a (M/M/1 ) : ( K/FIFO) system, find the average number of customers in the system.
10. A drive – in banking service is modeled as an M/M/1 queueing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99 percent of the time. How fast should the service rate be?
11. If people arrive to purchase cinema tickets at the average rate of 6 per minute ,it takes an average of 7.5 seconds to purchase a ticket . If a person arrives 2 minutes before the picture starts and it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket. Can he expect to be seated for the start of the picture?
12. Find the formula for $W_s$ and $W_q$ for the M/M/1/N queueing system.
13. For (M/M/C): (N/FIFO) model, write down the formula for (*a*) average number of customers in the queue. (*b*) average waiting time in the system.
14. Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join the queue.
15. What is the effective arrival rate in an (M/M/C ) : ( K/FIFO) queueing model?
16. If there are 2 servers in an infinite capacity Poisson queue system with $\lambda$= 10 *per hour* and $\mu$ = 15 *per hour,* what is the percentage of idle time for each server?
17. A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 *minutes* while the cashier can serve 10 customers in 5 *minutes.* Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find the
    i) average time a customer spends in the system
    ii) average time a customer waits before being served.
18. Write down the formulae for $P_0$ and $P_n$ in a Poisson queue system in the steady – state.
19. In a 3 server infinite capacity Poisson queue model if $\dfrac{\lambda}{\mu C} = \dfrac{2}{3}$, find $P_0$.
20. In a 3 server infinite capacity Poisson queue model if $\dfrac{\lambda}{c\,\mu} = \dfrac{2}{3}$ and $P_0 = \dfrac{1}{9}$, find the average number of customers in the queue and in the system.

**PART – B**

1. a) Customers arrive at a one – man barber shop according to a Poisson process with a mean interarrival time of 20 *minutes.* Customers spend an average of 15 *minutes* in the barber's chair. If an hour is used as a unit of time, then
   i) What is the probability that a customer need not wait for a haircut?
   ii) What is the expected number of customers in the barber shop and in the queue?
   iii) How much time can a customer expect to spend in the barber shop?
   iv) Find the average time that the customer spends in the queue
   v) The owner of the shop will provide another chair and hire another barber when a customer's average time in the shop exceeds 1.25 *hr*. By how much should the average rate of arrivals increase in order to justify a second barber?
   vi) Estimate the fraction of the day that the customer will be idle.
   vii) What is the probability that there will be more than 6 customers waiting for service?
   viii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair.
   ix) What is the probability that the waiting time (a) in the system (b) in the queue, is greater than 12 *minutes?*

   b) A petrol pump station has 2 pumps. The service times follows the exponential distribution with a mean of 4 *minutes* and cars arrive for service in a Poisson process at the rate of 10 cars *per hour.* Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle?

2. a) Assuming that customers arrive in a Poisson fashion to the counter at a supermarket at an average rate of 15 *per hour* and the service by the clerk has an exponential distribution, determine at what average rate must a clerk work in order to ensure a probability of 0.90 that the customer will not wait longer than 12 *minutes?*

   b) Suppose there are 3 typists in a typing pool. Each typist can type an average of 6 *letters/hr.* If the letters arrive to be typed at the rate of 15 *letter / hr,*
   i) what fraction of the time are all three typists busy?
   ii) what is the average number of letters waiting to be typed?
   iii) what is the probability that there is one letter in the system?
   iv) what is the average time a letter spends in the system ( waiting and being typed)?
   v) what is the probability a letter will take longer than 20 *minutes* waiting to be typed and being typed?
   vi) Suppose that each individual typist receives letters at the average rate of 5 / *hr* Assume each typist can type at the average rate of 6 letters / *hr*. What is the average time a letter spends in the system waiting and being typed?

3. a) A TV repairman finds that the time spend on his jobs has an exponential distribution with mean 30 *minutes.* If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 – *hour day.*
   i) what is the repairman's expected idle time in each day?
   ii) how many jobs are ahead of the average set just brought in?

   b) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average of 15 *per hour.* The length of service on these calls is approximately exponentially distributed with mean length 5 *minutes.*
   i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?
   ii) If the subscribers will wait and are serviced in turn, what is the expected waiting time?

4. a) On an average 96 patients per 24 hour day require the service of an emergency clinic. Also on an average, a patient requires 10 *minutes* of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 *minutes,* and that each minute of decrease in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patient?

   b) If for a period of 2 *hours* in the day ( 10 *am* to 12 *am*) trains arrive at the yard every 20 *minutes* but the service time continues to remain 36 *minutes*, calculate the following for the above said period:
   i) The probability that the yard is empty.
   ii) The average number of trains (average queue length) on the assumption that the line capacity of the yard is limited to 4 trains only.

5. a) Suppose people arrive to purchase tickets for a basketball game at the average rate of 4 *min.* It takes an average of 10 *seconds* to purchase a ticket. If a sports fan arrives 2 *min* before the game starts and if it takes exactly 1 ½ *min* to reach the correct seat after the fan purchased a ticket, then
   i) Can the sports fan expect to be seated for the start of the game?
   ii) What is the probability that the sports fan will be seated for the start of the game?
   iii) How early must the fan arrive in order to be 99% sure of being seated for the start of the game?

   b) The railway marshalling yard is sufficient only for trains (there being 11 lines, one of which is earmarked for the shunting engine to reverse itself from the crest of the hump to the rear of the train). Trains arrive at the rate of 25 trains per day, inter – arrival time and service time follow exponential with an average of 30 *minutes.* Determine the
   i) probability that the yard is empty.
   ii) average queue length.

6. a ) For the (M│M│1): ($G_D$ /∞/ ∞), derive the expression for $L_q$.

   b) A supermarket has two girls serving at the counter. The customers arrive in a Poisson fashion at the rate of 12 *per hour*. The service time for each customer is exponential with mean 6 *minutes.* Find the
   i) probability that an arriving customer has to wait for service.
   ii) average number of customers in the system, and
   iii) average time spent by a customer in the supermarket.

7. a) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more that 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.
   i) What is the probability that an arriving patient does not have to wait?
   ii) What is the expected waiting time until a patient is discharged from the clinic?

   b) A car servicing station has two bags where service can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with mean 6 *minutes.* Find the
   i) average number of cars in the service station
   ii) average number of cars in the system during the peak hours.
   iii) average waiting time a car spends in the system.
   iv) average number of cars per hour that cannot enter the station because of full capacity.

8. a) A bank has two tellers working on savings accounts. The first teller, handles withdrawals only. The second teller handles deposits only. It has been found that the service time distribution for

both deposits and withdrawals are exponential with mean service time 3 *min* / customer. Depositors are found to arrive in Poisson fashion throughout the day with a mean arrival rate of 16 / *hour.* Withdrawers also arrive in Poisson fashion with mean arrival rate of 14 *per hour*. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both withdrawals and deposits? What could be the effect if this could be accomplished by increasing the mean service time to 3.5 *minutes?*

b) A group of users in a computer browsing centre has 2 terminals. The average computing job requires 20 *min* of terminal time and each user requires some computation about once every half an hour. Assume that the arrival rate is Poisson and service rate is exponential and the group contains 6 users. Calculate the

i) average number of users waiting to use one of the terminals and in the computing job.
ii) total time lost by all the users per day when the centre is opened 12 *hrs/day.*

9. a) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy? What s the average number of letters waiting to be typed?(Assume Poisson arrival and exponential service times)

b) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the number of trains in the system. Also find the average waiting time of a new train coming into the yard. If the handling rate is reduced to half, what is the effect of the above results?

10. a) Derive $p_0, L_s, L_q, W_s, W_q$ *for* $(M/M/s):(\infty/FIFO)$ queueing model .

b) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 *kms.* down the river. Tankers arrive according to a Poisson process with a mean of 1 for every 2 *hours*. It takes for an unloading crew, on the average, 10 *hours* to unload a tanker, the unloading time follows an exponential distribution Determine.

i) how many tankers are at the port on the average?
ii) how long does a tanker spend at the port on the average?
iii) what is the average arrival rate at the overflow facility?

## UNIT - V - NON MARKOVIAN QUEUES AND QUEUE NETWORKS

### PART – A

1. Write down Pollaczek- Kinchin formula.
2. Define effective arrival rate with respect to an (M | M | 1): (GD / N/ ∞ ) queuing model**.**
3. For an M/G/1 model if $\lambda$=5 and $\mu$=6 min and $\sigma$=1/20, find the length of the queue.
4. An one man barber shop taken 25 mins to complete a heir cut. If customers arrive in a Poisson fashion at an average rate of 1 per 40 mins, find the average length of the queue.
5. Define a tandem queue.
6. What is a series queue with blocking?
7. A transfer line has two machines M1 and M2 with unlimited buffer space in between. Parts arrive t the transfer line at the rate of 1 part every 2 mins. The processing rates of M1 and M2 are 1 per min. and 2 per min. respectively. Find the average number of parts in M1.
8. Define an open Jackson network.
9. Write down the characteristics of an open Jackson network.
10. Define a closed Jackson network and state the modified flow equations.
11. Write down the traffic equations of an open Jackson network.

12. State the equivalence property of a queueing system.
13. State the arrival theorem in the study of Jackson network.
14. State the mean value analysis algorithm for single server network.
15. Define series queues.
16. Give any two examples for series queues.
17. Define a tendem queue.
18. Write classification of queueing networks.
19. Define a closed queueing network.
20. Distinguish between open and closed networks.

**PART – B**

1. a) Derive Pollaczek - Khinchin formula.
   b) In a heavy machine shop, the overhead crane is 75% utilized. Time study observations gave the average slinging time as 10.5 minutes with a standard deviation of 8.8 minutes. What is the average calling rate for the services of the crane and what is the average delay in getting service? If the average service time is cut to 8.0 minutes with a standard deviation of 6.0 minutes, how much reduction will occur on average in the delay of getting served?

2. a) Write the brief note on (i) Open queueing network and (ii) Closed queueing network.
   b) Automatic car wash facility operates with only one Bay. Cars arrive according to a Poisson process , with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 min, determine Ls, Lq, Ws and Wq.
3. a) In open Jackson network the following informations are given:

| | | | | $r_{ij}$ | | |
|---|---|---|---|---|---|---|
| Station | $C_j$ | $\mu_j$ | $r_j$ | i = 1 | i =2 | i =3 |
| 1 | 1 | 10 | 1 | 0 | 0.1 | 0.4 |
| 2 | 2 | 10 | 4 | 0.6 | 0 | 0.4 |
| 3 | 1 | 10 | 3 | 0.3 | 0.3 | 0 |

   Find (i) the joint probability for the number of customers in $1^{st}$ , $2^{nd}$ and $3^{rd}$ stations are 2,3,4 respectively.
   (ii) the expected number of customer in each station.
   (iii) the expected total number of customers in the system.
   (iv) the expected total waiting time in the system.
   b) A repair facility is shared by a large number of machines for repair. The facility has two sequential stations with respective rates of service 1 per hour and 3 per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by a two-station tandem queue. Find (i) the average number of customers in both stations, (ii) the average repair time, (iii) the probability that both service stations are idle.

4. a) In a computer programs for execution arrive according to Poisson law with a mean of 5 per minute. Assuming the system is busy, find $L_q$, $L_s$, $W_q$ , $W_s$ if the service time is (i) uniform between 8 and 12 sec. (ii) discrete with values 2,7 and 12 sec. and probabilities: 0.2, 0.5, 0.3
   b) A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer in the spends in the shop. Also, find the average time a customer must wait for service?

5. a) In super market during peak hours customers arrive according to a Poisson process at a mean rate of 40 per hour. A customer on the average takes 45 min to choose the food products and other articles that the customers needs. These times are exponentially distributed. The billing times are also exponentially distributed with a mean 4 min. For each counter (i) Find the minimum number of

counters required for billing during the peak hours. (ii) If the number of counters is one more than the minimum, how many will be in the queue? And how many will be in the supermarket?

b) There are two salesmen in a ration shop, one in charge of billing and receiving payment and the other in charge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop, that too when the billing clerk is free. The customer who has finished his billing job has to wait there until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rates of 3 and 2, find (i) the proportion of customers who enter the ration shop (ii) the average number of customers in shop and (iii) the average amount of time that an entering customer spends in the shop.

6. a) There are two service stations S1 and S2in a line with unlimited buffer space in between. Customers arrive t S1 at a rate of 1 per every 2 min. The service time rates of S1 and S2 are 1 and 2 per min. respectively. Find (i) the average number of customers at S1 and S2 (ii) The average waiting times at S1and S2 (iii) the total waiting time in the system.

   b) In a network of 3 service station 1,2, 3 customer arrive at 1,2,3 from outside in accordance with Poisson process having rate 5, 10, 15 res. The service time at the stations are exponential with respect rate 10, 50, 100, A customer completing service at station -1 is equally likely to (i) go to station 2 (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. (a) What is the average number customer in the system consisting of all the three stations? (b) What is the average time a customer spend in the system?

7. a) A repair facility is shared by a large number of machines for repair. The facility has two sequential stations with respective rates of service 1 per hour and 3 per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by a two-station tandem queue. Find (i) the average number of customers in both stations, (ii) the average repair time, (iii) the probability that both service stations are idle.

   b) Derive the formula for average length of the M/G/1 system.

8. a) In a charity clinic there are two doctors, one assistant doctor D1 and his senior doctor D2. The junior doctor tests and writes the case sheet and then sends to the senior for diagnosis and prescription of medicine. Only one patient is allowed to enter the clinic at a time due to paucity of space. A patient who has finished with D1 has to wait till the patient with D2 has finished. If patients arrive according to Poisson with rate 1 per hour and service times are independent and follow exponential with parameters 3 and 2, Find (i) the probability of a customer entering the clinic, (ii) the average number of customers in the clinic, (iii) the average time spent by a patient who entered the clinic.

   b) Consider a queuing system where arrivals according to a Poisson distribution with mean 5/hr. Find expected waiting time in the system if the service time distribution is Uniform from t = 5 min to t = 15 minutes.

9. a) A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with mean of 4 cars per hour and may wait in the factory's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for a car has uniform distribution between 8 and 12 minutes. Find (i) the average number of cars waiting in the parking lot (ii) the average waiting time of a car in the parking lot.

   b) Customers arrive at a service centre consisting of 2 service points S1 and S2 at a Poisson rate of 35/hour and form a queue at the entrance. On studying the situation at the centre, they decide to go to either S1 or S2 .The decision making takes on the average 30 seconds in an exponential fashion. Nearly 55% of the customers go to S1, that consists of 3 parallel servers and the rest go to S2, that consist of 7 parallel servers. The service times at S1, are exponential with a mean of 6 minutes and those at S2 with a mean of 20 minutes. About 2% of customers, on finishing service at S1 go to S2

and about 1% of customers, on finishing service at S2 go to S1. Find the average queue sizes in front of each node and the total average time a customer spends in the service centre.

10. a) In a departmental store, there are two sections namely grocery section and perishable section. Customers from outside arrive the G-section according to a poisson process at a mean rate of 10 per hour and they reach the p-section at a mean rate of 2 per hour. The service times at both the sections are exponentially distributed with parameters 15 and 12 respectively. On finishing the job in G-section, a customer is equally likely to go to the P-section or leave the store, where as a customer on finishing his job in the P-section will go to the G- section with probability 0.25 and leave the store otherwise. Assuming that there is only one salesman in each section, find (i) the probability that there are 3 customers in the G-section and 2 customers in the P-section, (ii) the average waiting time of a customer in the store.

b) A patient who goes to a single doctor clinic for a general check up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check up and the time taken for each phase is exponentially distributed. If the arrivals of the patients at the clinic are approximately Poisson at the average rate of 3 per hour, what is the average time spent by a patient (i) in the examination (ii) waiting in the clinic?